Derivation of LF when the rainfall is ignored.

The Leaching Fraction (LF) equation used in the calculation of Nitrate-nitrogen loading to the ground water is:

\[ LF = \frac{ET \cdot Cli \cdot 10^{-6} - Clc}{ET \cdot Clp \cdot 10^{-6} - Clc} \]

Where:
LF = Leaching Fraction.
ET = Seasonal evapotranspiration (kg/ha).
Cli = Chloride concentration in the irrigation water (mg/l).
Clp = Chloride concentration in the percolating water below the crop root zone (mg/l).
Clc = Amount of chloride taken up by the crop (kg/ha).

The equation is derived using the definition of leaching fraction and the mass balance equation for chloride:

The Leaching Fraction (LF) is defined as:

\[ LF = \frac{V_{p}}{V_{i}} \] \[ (1) \]

where:
Vp: Volume of percolating water below the crop root zone (l).
Vi: Volume of irrigation water (l).

The volume of irrigation water is:

\[ V_{i} = V_{p} + ET \] \[ (2) \]

where ET is seasonal evapotranspiration (l).

Substituting eq. 2 into eq. 1 results in eq. 3:

\[ LF = \frac{V_{p}}{V_{p} + ET} \] \[ (3) \]

**By Mass balance under steady state conditions, the chloride input equals the chloride output**

input = output

\[ Cli \cdot V_{i} = (Clp \cdot V_{p}) + Clc \] \[ (4) \]

Where:
Cli = Chloride concentration in the irrigation water (mg/l).
Vi = Volume of irrigation water (l).
Clp = Chloride concentration in the percolating water below the crop root zone (mg/l).
Vp = Volume of percolating water below the crop root zone (l).
Clc = Amount of chloride taken up by the crop (mg).

Substituting eq. 2 into eq. 4 and solving for Vp results in eq. 5:

\[ Cli \cdot (V_{p} + ET) = (Clp \cdot V_{p}) + Clc \]
\[ Cli \cdot V_{p} + Cli \cdot ET = Clp \cdot V_{p} + Clc \]
\[ Cli \cdot ET - Clc = Clp \cdot V_{p} - Cli \cdot V_{p} \]
\[ Cli \cdot ET - Clc = V_{p} (Clp - Cli) \]
\[ V_{p} = \frac{(Cli \cdot ET) - Clc}{Clp - Cli} \] \[ (5) \]
Solving eq. 3 for $V_p$ results in eq. 6

$$V_p = \frac{[LF \times ET]}{[1-LF]} ...... (6)$$

Substituting eq. 5 into eq. 6 and changing ET and Clc units to kg/ha and then solving for LF results in eq. 7

$$V_p = \frac{(LF \times ET)}{(1-LF)} = \frac{[(Cli \times ET) - Clc]}{[Clp-Cli]}$$

$$[Clp - Cli][LF \times ET] = [1-LF][(Cli \times ET) - Clc]$$

$$LF \times ET \times Clp - LF \times ET \times Cli = ET \times Cli - Clc - LF \times ET \times Cli + LF \times Clc$$

$$LF = \frac{ET \times Cli - Clc}{ET \times Clp - Clc} = \frac{ET \times Cli \times 10^{-6} - Clc}{ET \times Clp \times 10^{-6} - Clc}......(7)$$

Where:

- $LF$ = Leaching Fraction.
- $ET$ = Seasonal evapotranspiration (kg/ha).
- $Cli$ = Chloride concentration in the irrigation water (mg/l)
- $Clp$ = Chloride concentration in the percolating water below the crop root zone (mg/l).
- $Clc$ = Amount of chloride taken up by the crop (kg/ha).

**Leaching fraction calculation including rainfall**

$V_r$ = volume of rainfall

$Clr$ = chloride of rainfall

Volume of the irrigation plus rainfall is:

$$V_r+V_i = V_p+ET$$

$$V_i = V_p+ET-V_r ..... (8)$$

Mass Balance of the chloride is:

$$Cli \times V_i + Clr \times V_r = Clp \times V_p + Clc..... (9)$$

Substitute eq 8 into 9 and solve for $V_p$

$$Cli(V_p+ET-V_r)+(Clr\times V_r)=(Clp\times V_p)+Clc$$

$$Cli\times V_p+Cli\times ET-Cli\times V_r = Clr \times V_p + Clc$$

$$Cli\times V_p-Cli\times Clp = V_p+Cli\times Vr+Clr\times Vr-Cli\times Et$$

$$V_p=Cli-Cli\times Vr+Cli\times Clr+Clr\times Vr-Cli\times Et/ Cli-Clp......(10)$$

Take eq. 6 and substitute equation 10 into eq. 6

$$V_p= \frac{LF \times ET}{1-LF} ..... copy \ of \ eq.(6)$$

$$LF \times ET/1-LF = Clc-Cli\times Vr-Cli\times Vr-Cli\times Et/ Cli-Clp$$

$$:LF \times Et(Cli-Clp) = (1-LF)(Clc-Cli\times Vr+Clr\times Vr-Cli\times Et)$$

$$\frac{(LF \times ET \times Cli)}{(LF \times ET \times Clp)} = Clc-(LF \times Clc)-(Clr \times Vr)+(LF \times Clr \times Vr)+(Clr \times Vr)-(LF \times Cli \times Vr)-(Clr \times Et)+(LF \times Clp \times Et)$$

Simplify remove left and right side $Lr\times Et*CLI$

$$- (LF \times ET \times Clp) = Clc-(LF \times Clc)-(Clr \times Vr)+(LF \times Clr \times Vr)+(Clr \times Vr)-(LF \times Cli \times Vr)-(Clr \times Et)+(LF \times Clp \times Et)$$
move terms with Lf on left side
-(LF*ET*Clp)+(LF*Clc)-(LF*Clr*Vr)+(Lf*Cli*Vr)= Clc- (Clr*Vr)+((Cli*Vr)- (Cli*ET)

Lf(-ET*Clp+Clc- (Clr*Vr)+(Cli*Vr))= Clc+Vr(Cli-Clr)-Clr*Et

LF= Clc+Vr(Cli-Clr) - Cli*ET/ (Cli*Vr) - (Clr*Vr) -ET*Clp+Clc

Multiple numerator and denominator by - sign

LF= Cli*ET-Clc- Vr(Cli-Clr)/ Et* Clp - Clc -Vr*(Clr-Clr) ..... eq. 11

LF= Cli*ET10^-6-Clc- Vr(Cli-Clr)10^-6/ Et* Clp10^-6 - Clc -Vr*(Clr-Clr)10^-6 ..... eq. 12

ET and Vr and Clc in units of Kg/ha and Cl in units of (mg/l)